Multiscale Modeling of Magnetic Distribution of Ribbon Magnetic Cores

Hailin Li, Zuqi Tang, Shuhong Wang, and Jianguo Zhu

***Abstract*—The multiscale finite element method (MsFEM) combined with conventional finite element method (CFEM) is proposed to solve static magnetic field in the ribbon magnetic core with non-periodical corners considered. Firstly, a simple 2-dimensional electrostatic problem is used to introduce the MsFEM implementation process. The results are compared to analytical method, as well as conventional FEM. Then, an example of magneto-static problem is considered for a ribbon magnetic core built sheet by sheet as well as corners taken into consideration. Conventional FEM and MsFEM are used to compute the magneto-static field by adopting scalar magnetic potential. Both magnetic potential and magnetic flux density on a certain path are compared. It is shown that the results obtained by MsFEM agree well with the one from conventional FEM. Moreover, MsFEM combined with FEM is potentially a general strategy for multiscale modeling of ribbon magnetic cores with complex and non-periodical structures considered, like corners and T-joints, which can effectively reduce the computational cost.**

***Index Terms*—Magneto-static, multiscale finite element, multiscale modeling, ribbon magnetic core.**

I. Introduction

THE discretization of ribbon magnetic cores and laminated iron cores, i.e. by FEM, would lead to a prohibitively large system of equations since ribbon magnetic core and laminated iron core can be viewed as highly heterogeneous models. In three-dimensional (3-D) case, the issue of computational cost becomes even more severe. It is a huge difficulty for the modern computer to solve. Surely, it is far away from being a routine task for engineers in the design and performance analysis of electrical devices with such structure. To overcome this unpleasant fact, homogenization methods (HM) have been applied. In 2003, Dular *et al* adopted FEM combined with HM to study a three-dimensional 3-D magnetic field computation where the eddy currents in laminated stacks were taken into account [1]. In [2], [3], Gyselinck *et al* presented a novel time-domain HM for laminated iron cores in 3-D FE models for linear and nonlinear problems, respectively. In [4], Niyonzima *et al* adopted heterogeneous multiscale method to study laminated cores, which still needed homogenized material properties on representative volume elements (RVEs). Average quantities can be obtained by these computational HM methods. More often, it is desirable to capture the fine-scale effects, like the eddy current and magnetic distribution in each sheet of the ribbon magnetic core and iron core for loss calculation. In order to achieve this objective, researchers proposed some solutions. Duan *et al* proposed an improved extended finite element method (XFEM) for modeling electromagnetic devices with multiple nearby geometrical interfaces and discontinuities in electric fields [5]. Meanwhile, a ‘multiscale finite element method’ for the 2-D and 3-dimensional eddy current problem in iron laminates was put forwarded in [6], [7]. The mesh used for these two approaches are independent of geometries, and ‘correction functions’ (CFs) or ‘special functions’(SFs) are needed for basis functions construction of coarse elements first. For complex structures, like corners and T-joints of the transformers, it is difficult to find these CFs or SFs. It needs to find these CFs or SFs for each coarse element when non-periodical structures are of interest.

In this paper, we aimed to capture the magnetic distribution in small-scale of ribbon magnetic cores using another MsFEM which is firstly proposed by Hou and Wu for the elliptic problems in composite materials and porous media [8]. It has been used for fluid flow [9], groundwater flow [10], and mechanical analysis of heterogeneous materials [11]. This MsFEM was first introduced by Bottauscio and Manzin to solve eddy current problem, but with granular magnetic materials [12]. Compared with other multiscale modeling methods, MsFEM proposed by Hou and Wu do not use explicit expressions for basis functions construction. The basis functions are a set of numerical values on fine grids of each coarse element, which means that it can cope with basis functions construction of arbitrary geometries easily, such as T-joints and corners of an iron core. Hence, it is a general way to construct basis functions for elements with complex structures. In addition, results on small scale resolved by MsFEM can be as good as those solved by the CFEM. In this work, we developed this approach to model a magneto-static problem with application to magnetic core of a high-frequency transformer, and the magnetic distribution and flux density in each ribbon will be investigated. The organization of the paper is as follows: Firstly, a brief introduction to MsFEM and a simple case using the analytical method, FEM, and MsFEM are

Manuscript was submitted for review on 29, April, 2018.

Hailin Li is currently studying for a Ph.D. in Xi’an Jiaotong University (e-mail: hailinli@stu.xjtu.edu.cn).

Zuqi Tang is the Associate Professor in Electrical Engineering Department in University of Lille (e-mail: zuqi.tang@univ-lille.fr).

Shuhong Wang is currently a Professor with the School of Electrical Engineering, Xi’an Jiaotong University. (e-mail: shwang@mail.xjtu.edu.cn)

Jianguo Zhu is the Professor in University of Sydney (e-mail: jianguo.zhu@uts.edu.au).

Digital Object Identifier 10.30941/CESTEMS.2018.00054

examined. Then, application to magneto-static problem of MsFEM combined with FEM is considered in section III. The conclusion is given in section IV.

II. Mathematical Model

A. A brief introduction to MsFEM

The main idea of MsFEM is to construct multiscale basis functions, which capture fine-scale information within each coarse grid. The fine-scale information is then brought to coarse-scale through the coupling of global stiffness matrix. Thus, effects of fine-scale on the coarse-scale can be captured. Similar to XFEM, for our MsFEM, the mesh is independent of geometries, and a coarse grid may include several materials.

In general, there are three major steps for the implementation of MsFEM: multiscale basis function construction, global formulation, and downscaling analysis. To demonstrate the performance and implementation process of the MsFEM, a simple 2-D electrostatic problem is considered. The results obtained by MsFEM are compared with the analytical method, as well as the conventional FEM.

The governing equation and boundary conditions for the electrostatic problem are as follows:

$$\begin{cases} \dfrac{\partial}{\partial x}\left(\varepsilon\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial y}\left(\varepsilon\dfrac{\partial u}{\partial y}\right)=0, \\ u\big|_{x=0}=0, u\big|_{x=0.9}=1. \end{cases} \quad (1)$$

The simulation model is depicted in Fig.1. It consists of 3 coarse grids and 8 coarse-grid nodal points. Element ①, ③ contain only one material, while coarse element ②, included by the dashed line, contains two kinds of materials. Material property in light orange color is $\varepsilon_1=1$, while in the yellow region $\varepsilon_2=5$. Multiscale basis function construction in coarse grid ② will be detailed as follows:

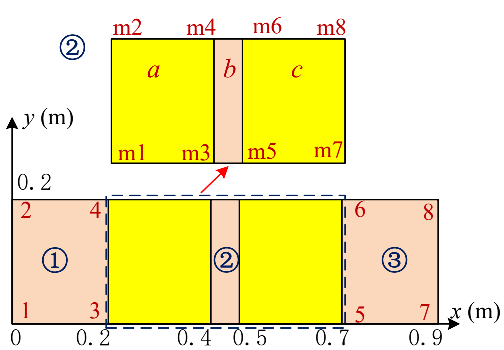


Fig.1. Coarse grids of the MsFEM.

1. Multiscale basis function construction

Multiscale basis functions satisfy the following local boundary problems,

$$\frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\phi_i^E}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\phi_i^E}{\partial y}\right)=0 \text{ in coarse grid } E. \quad (2)$$

Normally,

$$\begin{cases} \phi_i^E\left(x_j\right)=1, \text{ if } i=j, \\ \phi_i^E\left(x_j\right)=0, \text{ if } i\neq j. \end{cases} \quad (3)$$

Here, E is coarse grid ②. Multiscale basis functions can be obtained by solving the local boundary problems with specified boundary conditions. To be noticed, the boundary conditions for basis constructions are vital. Here, the oscillation boundary condition is used.

Firstly, coarse grid ② is refined into 3 finer grids, a, b, c, as shown in the upper part of Fig.1. Then, oscillation boundary conditions are used for multiscale basis function construction of coarse-grid nodes, namely 3, 5, 6, 4. For a general coarse grid E, oscillation boundary conditions for coarse-grid node i (i=3, 4, 5, 6) are shown in Fig.2. Here, "the boundary condition" value is something has the form like "u=g", and g can be piecewise functions or g=0 on Γ_{ij} (i, j=3, 4, 5, 6).

The boundary condition for the multiscale basis of coarse grid E on Γ_{im} is g_i^E and the function g_i^E is given by [7],

$$g_i^E\left(x\right)\Big|_{\Gamma_{ij}}=\int_x^{x_j}\frac{\mathrm{d}x}{K(x)}\Bigg/\int_{x_i}^{x_j}\frac{\mathrm{d}x}{K(x)} \quad (4)$$

where $K(x)$ is function of material property on the boundary.

With (4), the boundary condition on Γ_{35} for node i=3 of ② is,

$$\begin{matrix} & \text{m1} & \text{m2} & \text{m3} & \text{m4} & \text{m5} & \text{m6} & \text{m7} & \text{m8} \end{matrix}$$

$$g_3^{②}\left(x\right)\Big|_{\Gamma_{35}}=\begin{bmatrix}1 & 0 & \frac{7}{9} & 0 & \frac{2}{9} & 0 & 0 & 0\end{bmatrix}.$$

where m1-m8 are fine-grid nodal points. Boundary conditions are equal to the multiscale basis values in this case because there are no inside-nodes of coarse grid ②. Boundary conditions on other coarse-grid nodes are obtained similarly.

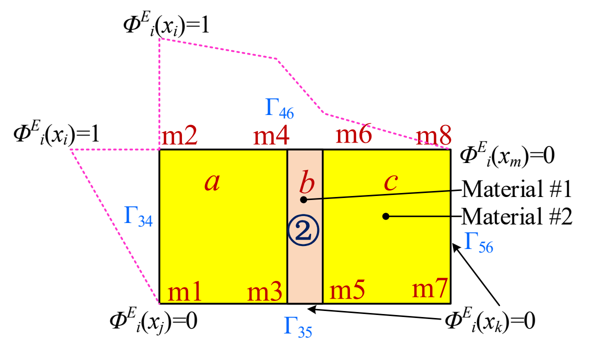


Fig.2. Oscillation boundary condition for coarse grid E.

2. Global formulation

Element in stiffness matrix $\boldsymbol{K}^E$ of coarse grid E is given,

$$K_{i,j}^E=\sum_{e=1}^{N_{element,E}}\int_{\Omega_e}\varepsilon^e\left(\begin{aligned}&\sum_{k=1}^{N_{node,e}^E}\frac{\partial N_k}{\partial x}\phi_i^{E,\mathrm{I}(e,k)}\bullet\sum_{l=1}^{N_{node,e}^E}\frac{\partial N_l}{\partial x}\phi_j^{E,\mathrm{I}(e,k)}\\&+\sum_{k=1}^{N_{node,e}^E}\frac{\partial N_k}{\partial y}\phi_i^{E,\mathrm{I}(e,k)}\bullet\sum_{l=1}^{N_{node,e}^E}\frac{\partial N_l}{\partial y}\phi_j^{E,\mathrm{I}(e,k)}\end{aligned}\right)\mathrm{d}x\mathrm{d}y \quad (5)$$

where E the index of coarse grid, here E is only for ② for our case; i, j the index of the node in the coarse grid E, here as E is the ② coarse grid, i, j = 3, 4, 5, 6; $N_{element,E}$ the number of refine grids in the coarse grid E, i.e here $N_{element,E}$ =3, namely for three grids: a, b and c; $N^E_{node,e}$ the number of node in the e-th refine element in the coarse grid E, i.e. 4 nodes for coarse grid ② here; ε^e in the refine element e; N_k, N_l the standard bilinear basis function of the conventional FEM associated to k-th or l-th node in the refined mesh; $\mathbf{I}(e, k)$ represents the index of k-th node of the e-th refined mesh in the coarse grid E. For example, when e=1, we consider the a refined grid, the four nodes are m_1, m_3, m_4 and m_2.

With (5), the stiffness matrix of coarse grid ② is,

$$K^{②} = \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 4.5617 & -4.0062 & 0.6883 & -1.2438 \\ -4.0062 & 4.5617 & -1.2438 & 0.6883 \\ 0.6883 & -1.2438 & 4.5617 & -4.0062 \\ -1.2438 & 0.6883 & -4.0062 & 4.5617 \end{bmatrix} \end{array}$$

where the number 3-6, on the top and the left side is the index of coarse-grid node.

Meanwhile, the stiffness matrices of coarse grid ①, ③ can be obtained by the conventional FEM.

3. Downscaling analysis

The solution on fine grids can be obtained by a linear combination of the multiscale basis function, as shown below.

$$u = \phi_i^E u_i + \phi_j^E u_j + \phi_k^E u_k + \phi_m^E u_m \tag{6}$$

Results calculated by analytical method, FEM and MsFEM are summarized in Table I. From this table, it is obvious that the result solved by MsFEM agrees well with analytical method and FEM. Validation of MsFEM can be proved.

TABLE I

RESULT COMPARISON OF ANALYTICAL METHOD, FEM, MSFEM

Coordinate	Analytical	FEM	MsFEM
(0.2,0)	0.3448	0.3448	0.3448
(0.2,0.2)	0.3448	0.3448	0.3448
(0.4,0)	0.4138	0.4138	0.4138
(0.4,0.2)	0.4138	0.4138	0.4138
(0.5,0)	0.5862	0.5862	0.5862
(0.5,0.2)	0.5862	0.5862	0.5862
(0.7,0)	0.6552	0.6552	0.6552
(0.7,0.2)	0.6552	0.6552	0.6552

Pseudo-code

A simple pseudo-code below outlines the implementation of MsFEM. It is obvious that this method can be realized very easily within the existing finite element code.

Algorithm:

Set up the coarse grid configuration, and obtain the fine grids information of each coarse grid
For each coarse grid E, do
-For each vertex i
-Get ϕ_i^E and boundary conditions
-End for
End do
Assemble macro stiffness matrix of coarse grid
Assemble macro load vector
Solve the global formulation
Down scaling analysis.

III. APPLICATION IN MAGNETO-STATIC PROBLEMS

A. Mathematical model

A magneto-static problem in 2-D is considered, and the governing equation of magneto-magnetic field is given by,

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu_r\mu_0}\frac{\partial A_z}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\mu_r\mu_0}\frac{\partial A_z}{\partial y}\right) = -j_s\,. \tag{7}$$

Boundary conditions are

$$A_z = 0, \text{ on } S_1, \ S_2, \tag{8}$$

and natural Neumann conditions on S_3, S_4.

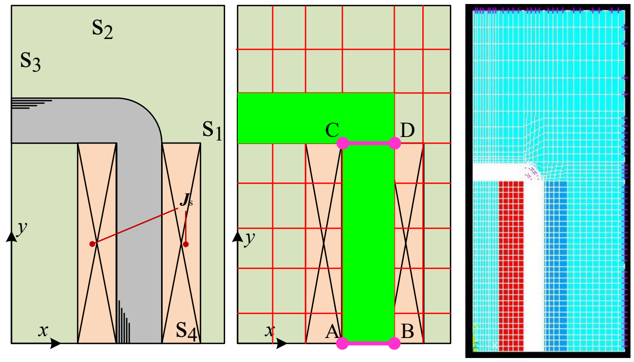


Fig.3. (a) Schematic of the simulation model, (b) coarse grids for MsFEM, (c) mesh grids for FEM.

The simulation model is illustrated in Fig.3 (a), and it is 1/4 part of the magnetic core. The ribbon magnetic core consists of 10 sheets. The thickness of each sheet is 0.35mm, and the insulation coating of each sheet is 0.0065mm. Current density j_s=12732.4 A/m^2. Relative magnetic permeability μ_r is 10000 for each sheet, and μ_r =1 for the air and the insulation, μ_0 is vacuum permeability.

The mesh for MsFEM, shown in Fig.3 (b), consists of 48 coarse grids, 63 coarse-grid nodes. The mesh, depicted in Fig.3(c), is for the conventional FEM. MsFEM combined with FEM is proposed to solve this problem. For the green-region coarse grids, multiscale basis functions are constructed following the procedures of section II. For other coarse grids, basis constructions follow the conventional FEM procedures, by adopting the bilinear basis function. Oscillation boundary conditions are used for multiscale basis construction. Elements of stiffness matrix are obtained by (5). Results on fine grids can be calculated by (6).

B. Results and discussions

Multiscale basis functions for coarse grid containing path A-B are illustrated in Fig.4. Obviously, they are different from the conventional polynomial basis functions.

Result data of the green region shown in Fig.3 (b) are extracted. Magnetic potentials calculated by conventional FEM and MsFEM are drawn in Fig.5 and Fig.6, respectively. Result solved by MsFEM agrees quite well with conventional FEM by comparison. Magnetic potential along path A-B, depicted in Fig.3 (b), is shown in Fig.7. From Fig.8, the max relative error, solved by (9), is -0.002% of this path.

$$\text{Relative error} = \frac{A_{z,\text{FEM}} - A_{z,\text{MsFEM}}}{A_{z,\text{FEM}}} \times 100\% \tag{9}$$

Magnetic flux density along path A-B is given in Fig.9. There is a deviation between the MsFEM and FEM. The max relative error is 1.08%, which is acceptable. In addition, magnetic flux density along path C-D, depicted in Fig.10, for the corner part of the core are examined to demonstrate accuracy for its potential in arbitrary geometry modeling and calculation such as T-joints and corners. The maximum relative error is 4.75%.

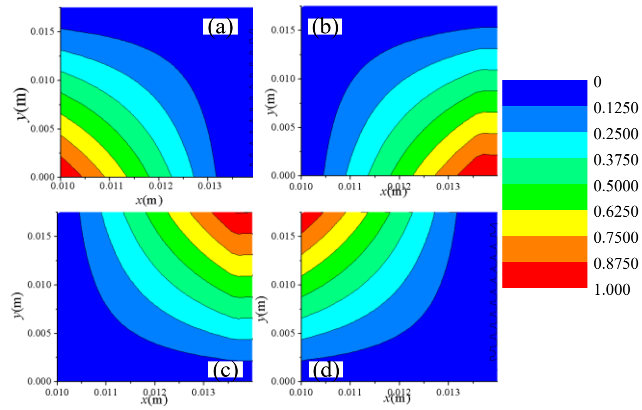


Fig.4. Multiscale basis for coarse grid containing path A-B.

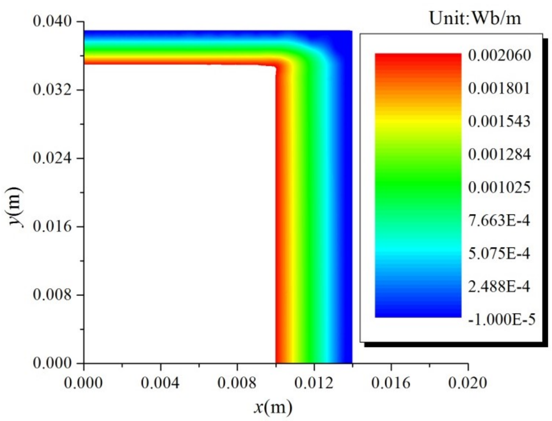


Fig.5. Magnetic potential distribution calculated by FEM.

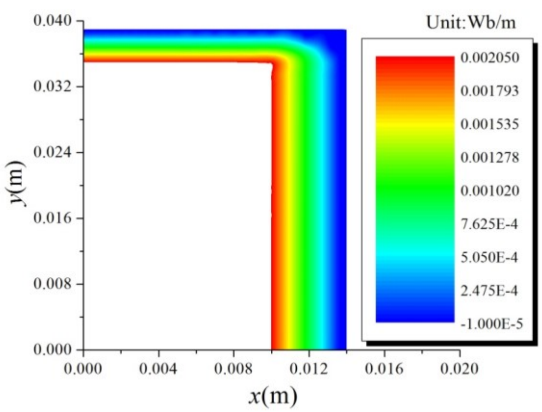


Fig.6. Magnetic potential distribution calculated by MsFEM.

Computation burden comparison of MsFEM and FEM are shown in Table II. It is found that equation systems obtained by using MsFEM is much smaller. The solving scale of MsFEM is 1/55.7 times as large of FEM. What is more, the normalized multiplication times for stiffness matrix formulation is 0.6284 by adopting the MsFEM combined with FEM. Obviously, MsFEM combined with FEM could significantly reduce the computation burden while the accuracy is maintained.

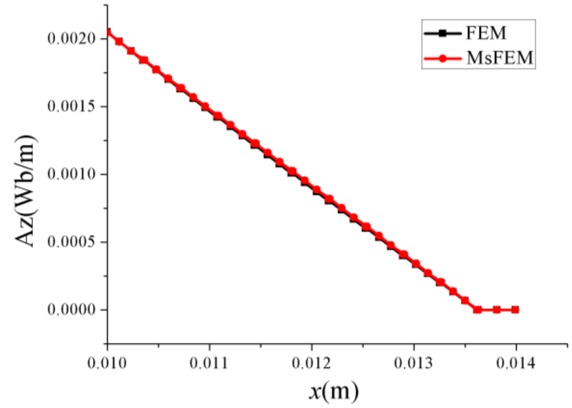


Fig.7. Magnetic potential on path A-B solved by FEM and MsFEM.

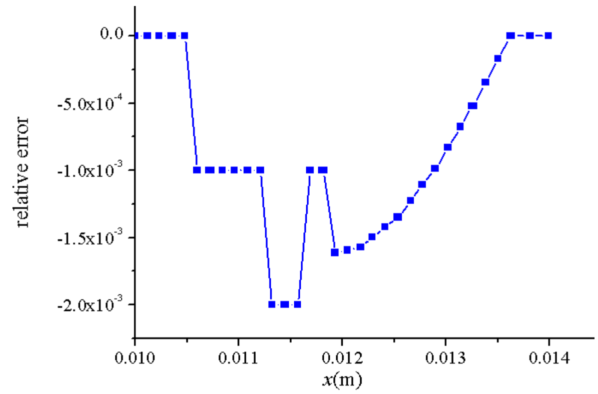


Fig.8. Relative error along path A-B.

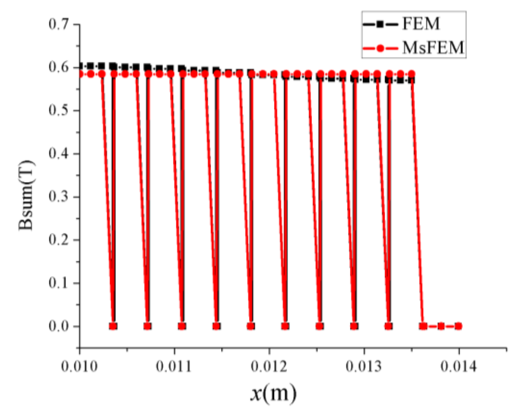


Fig.9. Magnetic flux density on path A-B computed by FEM and MsFEM.

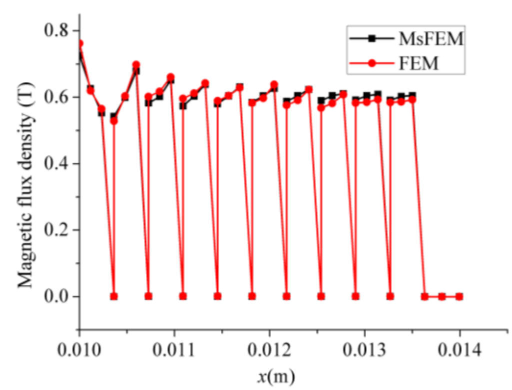


Fig.10. Magnetic flux density on path C-D computed by FEM and MsFEM.

IV. Conclusion

MsFEM combined with conventional FEM is proposed to solve magneto-static problem of ribbon magnetic core. Firstly, validation of MsFEM is examined by a simple case. Then, a magneto-static problem considered. By comparison the computation cost, it can be concluded that MsFEM combined with FEM can alleviate the computational burden and requirement of hardware for ribbon magnetic core's simulation effectively. What is more, it is a general and flexible method since it be implemented easily within the existing finite element code.

TABLE II
COMPUTATION BURDEN COMPARISON OF MsFEM AND FEM

	FEM	MsFEM
Element number	3396	48
Node number	3510	63
Multiplication times (normalized)	1	0.6284

References

[1] P. Dular, J. Gyselinck, C. Geuzaine, N. Sadowski, and J. Bastos, "A 3-D magnetic vector potential formulation taking eddy currents in lamination stacks into account," *IEEE Trans. Magn.*, vol. 39, pp. 1424–1427, Mar. 2003.

[2] J. Gyselinck, R. Sabariego, and P. Dular, "A nonlinear time-domain homogenization technique for laminated iron cores in three-dimensional finite-element models," *IEEE Trans. Magn.*, vol. 42, pp. 763–766, Apr., 2006.

[3] J. Gyselinck, P. Dular. "A time-domain homogenization technique for laminated iron cores in 3-D finite-element models," *IEEE Trans. Magn.*, vol. 40, pp. 856-859, Feb. 2004.

[4] I. Niyonzima, R. V. Sabariego，P. Dular，C. Geuzaine, "Computational homogenization for laminated ferromagnetic cores in magnetodynamics," *IEEE Trans. on Magn.*, vol. 49, no. 5, pp. 2049-2052, 2013.

[5] N. Duan, W. Xu, S. Wang, J. Zhu, and Y. Guo, "An improved XFEM with multiple high-order enrichment functions and low-order meshing elements for field analysis of electromagnetic devices with multiple nearby geometrical interfaces," *IEEE Trans. on Magn.*, vol. 51, pp. 1-4, Mar. 2015.

[6] K. Hannukainen, J. Schöberl, "Two-scale homogenization of the nonlinear eddy current problem with FEM," *IEEE Trans. on Magn.*, vol.50, pp.413-416, Feb., 2014.

[7] K. Hollaus, J. Schöberl, "Multi-scale FEM and magnetic vector potential a for 3D eddy currents in laminated media," *Compel International Journal of Computations & Mathematics in Electrical* vol. 34, pp.1598-1608, 2015.

[8] T. Y Hou, X. Wu, "A multiscale finite element method for elliptic problems in composite materials and porous media," *Journal of Computational Physics*, vol. 134, pp.169-189, 1997.

[9] T. Y Hou, "Multiscale modelling and computation of fluid flow," *International Journal for Numerical Methods in Fluids,* vol. 47, pp. 707-719, 2010.

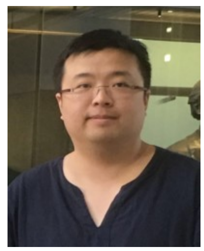
[10] Q. Xue, S. Ye, "Application of multi-scale finite element method to simulation of groundwater flow," *Journal of hydraulic engineering*, vol. 35, pp.07-13, 2004.

[11] H. Zhang, J. Lü, Z. Fu, "Extended multiscale finite element method for mechanical analysis of heterogeneous materials," *Acta Mechanica Sinica,* vol. 26, pp. 899-920, 2010.

[12] O. Bottauscio, A. Manzin, "Comparison of multiscale models for eddy current computation in granular magnetic materials," *Journal of Computational Physics*, vol. 253, pp.1-17, 2013.

Hailin Li was born in Hubei, China, in 1988. He received his B.E. from the Henan Polytechnic University, Henan, China, in 2011; M.E. from Shenyang University of Technology, Liaoning, China, in 2014.

He is currently studying for a Ph.D. in Xi'an Jiaotong University, Shaanxi, China, all in electrical engineering. His major research interest is numerical analysis of electromagnetic field and multi-physics coupling.



Zuqi Tang received the B.S. (2007) in mathematics from Wuhan University, China, the M.S. (2009) in applied mathematics, and the Ph.D. (2012) in electrical engineering in the University of Lille. From 2017, he is the Associate Professor in Electrical Engineering Department in University of Lille.

His current research is focused on the numerical modeling and analysis in computational electromagnetism, as well as coupled multi-physical problems.

Shuhong Wang (M'11-SM'13) was born in Shaanxi, China, in 1968. He received the B.E., M.E., and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, in 1990, 1993, and 2002, respectively, all in electrical engineering.

He is currently a Professor with the School of Electrical Engineering, Xi'an Jiaotong University.His research interests include numerical analysis of electromagnetic field, and design and optimization of electromagnetic devices.

Jianguo Zhu (S'93–M'96–SM'03) Prof. J.G. Zhu received his BE in 1982 from Jiangsu Institute of Technology, ME in 1987 from Shanghai University of Technology, China, and PhD in 1995 from University of Technology, Sydney (UTS), Australia, all in electrical engineering.

From 1982 to 1990, he was employed by Jiangsu Institute of Technology, China, as an associate lecturer and later on lecturer. In 1994, he was appointed a lecturer in School of Electrical Engineering at UTS. He was promoted to full professor of electrical engineering in 2004. Professor Zhu joined the University of Sydney in 2018. His current research interests include electromagnetics, magnetic properties of materials, electrical machines and drives, power electronics, and green energy systems.